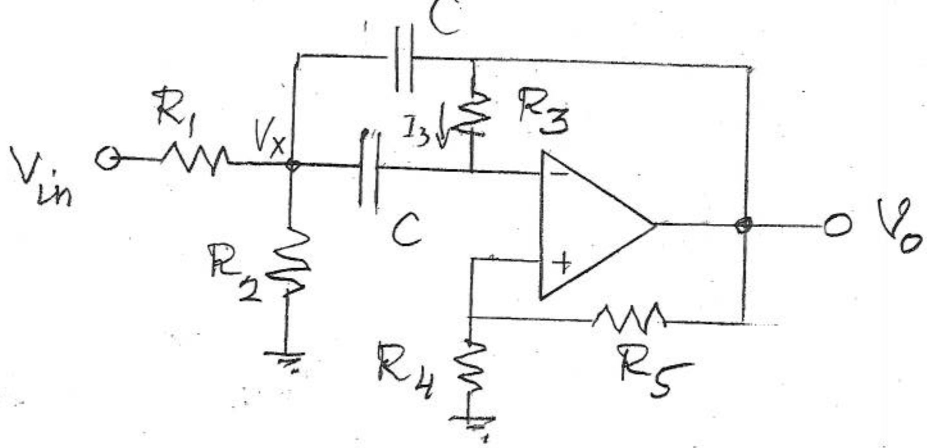
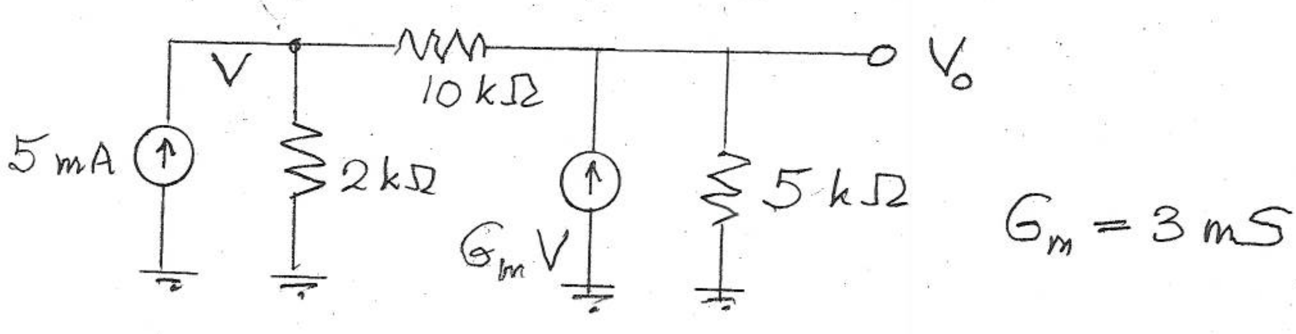
FINAL EXAMINATION
Dec. 8, 2008, 9:30 - 11:20 am

ECE 580
Prof. G. C. Temes

1. Analyze the active filter shown below. Find its transfer function
   H(s) = Vo(s)/Vin(s).



2. Find the sensitivity of Vo to variations of the transconductance Gm in the circuit shown, using the adjoint network method.



3. (Easy) Prove that the phase shift of a lumped linear two-port is an odd function of the frequency.

or

3. (Hard!) What is the asymptotic behavior of the group delay of a lumped linear two-port at very high frequencies? Why?

1' 10"

1. $I_3 = \left(V_o - V_o \cdot \frac{R_4}{R_4+R_5}\right) \cdot \frac{1}{R_3} = V_o \frac{R_5}{(R_4+R_5)R_3}$

$\therefore V_x = V_o - I_3 \cdot \left(R_3 + \frac{1}{sC}\right)$

$$= V_o - V_o \cdot \frac{R_5}{(R_4+R_5)R_3} \cdot \left(R_3 + \frac{1}{sC}\right) = V_o \cdot \frac{R_4R_3 - \frac{R_5}{sC}}{(R_4+R_5)R_3}$$

Nodal eq. at V_x

$$V_x \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + 2sC\right) - V_{in} \cdot \frac{1}{R_1} - V_o \cdot sC - V_o \cdot \frac{R_4}{R_4+R_5} sC = 0$$

$$\therefore V_{in} \frac{1}{R_1} = V_o \cdot \left[\frac{R_4R_3 - \frac{R_5}{sC}}{(R_4+R_5)R_3} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + 2sC\right) - sC \cdot \frac{2R_4+R_5}{R_4+R_5}\right] = 0$$

$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{R_1}}{\frac{R_4R_3 - \frac{R_5}{sC}}{(R_4+R_5)R_3} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + 2sC\right) - sC \frac{2R_4+R_5}{R_4+R_5}}$$

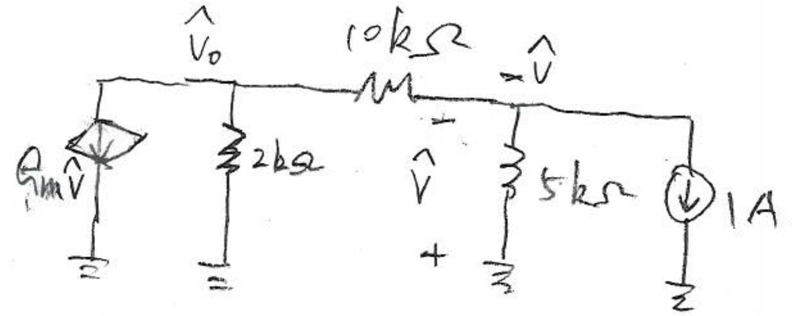
$$= \frac{\frac{1}{R_1}}{\frac{(R_4R_3 - R_5/sC)\left(\frac{1}{R_1} + \frac{1}{R_2} + 2sC\right) - sC(2R_4+R_5)R_3}{(R_4+R_5)R_3}}$$

$$= \frac{(R_4+R_5)R_3}{R_1\left[(R_4R_3 - R_5/sC)\left(\frac{1}{R_1} + \frac{1}{R_2} + 2sC\right) - sC(2R_4+R_5)R_3\right]}$$

$$= \frac{a_1 s}{b_2 s^2 + b_1 s + b_0}$$

$$= \frac{(R_4+R_5)R_3 C s}{-R_1R_3R_5C^2 s^2 + R_1\left[R_3R_4C\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - 2R_5C\right]s - R_1R_5\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

(2) $\hat{N}$:



Solve N:

$$\begin{cases} V \cdot \left(\frac{1}{2} + \frac{1}{10}\right) - V_0 \cdot \frac{1}{10} = 5 \\ V_0 \cdot \left(\frac{1}{10} + \frac{1}{5}\right) - V \cdot \frac{1}{10} = G_m \cdot V \end{cases} \Rightarrow V = -11.5V$$

Solve $\hat{N}$:

$$\begin{cases} -\hat{V} \cdot \left(\frac{1}{10} + \frac{1}{5}\right) - \hat{V}_0 \frac{1}{10} = -1000 \\ \hat{V}_0 \cdot \left(\frac{1}{10} + \frac{1}{2}\right) - (-\hat{V}) \frac{1}{10} = -G_m \hat{V} \end{cases} \Rightarrow \hat{V} = -4615 V$$

$\therefore \frac{\partial V_0}{\partial G_m} = V \cdot \hat{V} = 53 \; V/mS$

~~12-39~~ 3 (easy)

Since $H(s)$ is real rational, it can be written separately in even part and odd part

$$H(s) = H_e(s) + H_o(s) = P_e(s) + sQ_e(s)$$

$$= R_1(s^2) + s\,R_2(s^2)$$

For $s = j\omega$

$$H(j\omega) = R_1(-\omega^2) + j\omega\,R_2(-\omega^2)$$

$$\beta(\omega) = \tan^{-1} \frac{\omega R_2(-\omega^2)}{R_1(-\omega^2)} = \tan^{-1} U(\omega)$$

$U(\omega) = \dfrac{\omega R_2(-\omega^2)}{R_1(-\omega^2)}$ is an odd function.

Also $\theta(x) = \tan^{-1} x$ is an odd function.

$\beta(\omega)$ is obviously an odd function;

i.e. $\beta(-\omega) = -\beta(\omega)$

Si

E

c

R

12-41

For

a_0^n

a_0^r

(2n

For

a

~~19-51~~ 3 (hard)

If we write $H(s)$ as $H(s) = \dfrac{N(s)}{D(s)}$,

$$F(s) = T_g(\omega)\Big|_{\omega = s/j} = Ev\left[\frac{1}{H(s)}\frac{dH(s)}{ds}\right]$$

$$= Ev\left[\frac{D(s)}{N(s)}\,\frac{N'(s)D(s) + D'(s)N(s)}{D^2(s)}\right]$$

$$= Ev\left[\frac{N'(s)}{N(s)} + \frac{D'(s)}{D(s)}\right] = \frac{1}{2}\left[\frac{N'(s)}{N(s)} + \frac{N'(-s)}{N(-s)} + \frac{D'(s)}{D(s)} + \frac{D'(-s)}{D(-s)}\right]$$

$$= \frac{1}{2}\left[\frac{N'(s)N(-s) + N'(-s)N(s)}{N(s)\,N(-s)} + \frac{D'(s)D(-s) + D'(-s)D(s)}{D(s)\,D(-s)}\right]$$

Considering the numerator of the first term, $N'(s)\,N(-s) + N'(-s)\,N(s)$, the highest-degree terms are cancelled. Therefore the numerator is lower by 2 in degree than the denominator. Same argument goes to second part. Therefore $T_g(\omega) \to 0$ as const. $/\,\omega^2$ as $\omega \to \infty$.